

The holographic superconductors in higher-dimensional AdS soliton

Chong Oh Lee

*Department of Physics, Kunsan National University,
Kunsan 573-701, Korea
cohlee@kunsan.ac.kr*

Abstract

We explore the behaviors of the holographic superconductors at zero temperature for a charged scalar field coupled to a Maxwell field in higher-dimensional AdS soliton spacetime via analytical way. In the probe limit, we obtain the critical chemical potentials increase linearly as a total dimension d grows up. We find that the critical exponent for condensation operator is obtained as $1/2$ independently of d , and the charge density is linearly related to the chemical potential near the critical point. Furthermore, we consider a slightly generalized setup the Einstein-Power-Maxwell field theory, and find that the critical exponent for condensation operator is given as $1/(4 - 2n)$ in terms of a power parameter n of the Power-Maxwell field, and the charge density is proportional to the chemical potential to the power of $1/(2 - n)$.

1 Introduction

Nonlinear theory of electrodynamics has been suggested in Ref. [1] in search for an improvement over Maxwell theory with a infinite electrostatic self-energy of a point, and its extended form has been obtained in Ref. [2]. It has been found in Ref. [3] through investigation of transition to state of virtual charged particle in quantum electrodynamics. It has been also studied in gravity theory. For example, black hole solutions are obtained from nonlinear electrodynamics minimally coupled to gravity for a static and spherical symmetric spacetime [4], and by nonlinear electrodynamics with power-law function [5].

On the other hand, for asymptotically AdS spacetime, it is of interest to attempt to study the phase transition in the model for holographic superconductors [6, 7] since it allows new predictions through exploring the proposed AdS/CFT correspondence [8, 9, 10], which relates a gravitational theory on asymptotically in the bulk to a conformal field theory in the boundary. Their behaviors have been explored by a gravitational theory of a charged scalar field coupled to a Maxwell field [11, 12, 13]. The gravity model of the holographic superconductor has revived many investigations for their potential applications along these directions [14]-[29]. A few phase transition studies in a Stueckelberg form have been carried out [30]-[37]. Furthermore a superconducting phase dual to the AdS soliton configuration is interesting case [33]-[37] since the AdS black hole in the Poincaré coordinate can exhibit a phase transition to the AdS soliton even if the AdS black hole and the AdS soliton have the same boundary topology in asymptotically AdS spacetimes [38].

Even if the model for holographic superconductors is well established in four- and five-dimensional spacetime it is less explored in higher-dimensional spacetime. Thus, one intriguing question is higher-dimensional behaviors for holographic superconductors. Another is how they are affected from the Power-Maxwell field since they are governed by the gravity theory with electric field coupled to the charged scalar field.

In this paper we consider the Einstein-Maxwell field theory in higher-dimensional AdS soliton and find the critical exponent for condensation operator is $1/2$ independently of d in the limit of probe at zero temperature, and the charge density is directly proportional to the chemical potential.

The paper is organized as follows: In the next section we investigate the model for holographic superconductors. We obtain the critical chemical potentials for various dimensions of operators in d -dimensional spacetime, and the relations between the charge density and the chemical potential near the critical point. In the last section we give our conclusion.

2 Holographic Duality in the AdS soliton background

In this section, we will construct the phase transition model for the Einstein-Power-Maxwell field theory in the AdS soliton background.

Considering a superconductor dual to a AdS soliton configuration in the probe limit, the line element of d -dimensional AdS soliton is given by [33, 39, 40]

$$ds^2 = \frac{dr^2}{f(r)} + \frac{r^2}{L^2}(-dt^2 + h_{ij}dx^i dx^j) + f(r)d\eta^2, \quad (2.1)$$

with

$$f(r) = \frac{r^2}{L^2} \left(1 - \frac{L^{d-1}r_0^{d-1}}{r^{d-1}} \right), \quad (2.2)$$

where L is AdS radius and r_0 is the tip of soliton. One must impose the periodicity $\eta \sim \eta + \frac{\pi}{r_0}$ to avoid a conical singularity [41]. The d -dimensional Power-Maxwell-scalar action with negative cosmological constant is

$$S = \int d^d x \sqrt{-g} \quad \left\{ R - 2\Lambda - \alpha(F_{\mu\nu}F^{\mu\nu})^n - \partial_\mu \Psi \partial^\mu \Psi - m^2 \Psi^2 \right. \\ \left. - \Psi^2 (\partial_\mu \Phi - q A_\mu)(\partial^\mu \Phi - q A^\mu) \right\}, \quad (2.3)$$

where g denotes the determinant of the metric, R the Ricci scalar, and $\Lambda = (d-1)(d-2)/L^2$ the cosmological constant. $F^{\mu\nu}$ is the strength of the Power-Maxwell (PM) field $F = dA$, the complex scalar field Ψ , the coupling constant α , and the power of PM field n . We may take the solutions of r only,

$$A = \phi(r)dt, \quad \Psi = |\Psi| = \psi(r), \quad (2.4)$$

and impose the gauge choice $\Phi = 0$, and set $L = 1$ and $q = 1$ through appropriately scaling symmetries in as [22]. Then the equations of motion are given by

$$\ddot{\psi} + \left(\frac{\dot{f}}{f} + \frac{d-2}{r} \right) \dot{\psi} + \left(\frac{r^2 \phi^2}{f} - \frac{m^2}{f} \right) \psi = 0, \quad (2.5)$$

$$\ddot{\phi} + \left\{ \frac{\dot{f}}{f} + \left(\frac{d-4}{2n-1} \right) \frac{1}{r} \right\} \dot{\phi} + \frac{1}{\alpha n(2n-1)(-2)^n} \frac{\psi^2 \phi}{\dot{\phi}^{2(n-1)} f} = 0, \quad (2.6)$$

which leads to

$$\psi'' + \left(\frac{f'}{f} - \frac{d-4}{z} \right) \psi' + \frac{r_0^2}{z^4} \left(\frac{z^2 \phi^2}{f} - \frac{m^2}{f} \right) \psi = 0, \quad (2.7)$$

$$\phi'' + \left\{ \frac{f'}{f} - \left(\frac{d-4}{2n-1} - 2 \right) \frac{1}{z} \right\} \phi' - \frac{r_0^{2n}}{\alpha n(2n-1)(-1)^{3n+1} 2^n z^{4n}} \frac{\psi^2 \phi}{(\phi')^{2(n-1)} f} = 0, \quad (2.8)$$

by introducing a new coordinate $z = r_0/r$. Here a dot denotes the derivative with respect to r and a prime is the derivative with respect to z .

In order to solve the above equations, one needs to impose boundary condition at the tip $z = 1$ ($r = r_0$) and one at the origin $z = 0$ ($r = \infty$). Thus, at the tip one can do the expansion

$$\psi(z) = a_1 + a_2(z-1) + a_3(z-1)^2 + \dots, \quad (2.9)$$

$$\phi(z) = b_1 + b_2(z-1) + b_3(z-1)^2 + \dots, \quad (2.10)$$

$$f(z) = c_2(z-1) + \dots, \quad (2.11)$$

whose solutions behave as

$$\psi(z=1) = a_1, \quad (2.12)$$

$$\phi(z=1) = b_1, \quad (2.13)$$

where a_1 and b_1 are constants. Since one can set $r_0 = 1$ through appropriately scaling symmetries in as [22], at the origin, the solutions behave as

$$\psi = z^{\lambda_-} \psi_- + z^{\lambda_+} \psi_+, \quad (2.14)$$

$$\phi = \mu - \rho z^{(d-2)/(2n-1)-1}, \quad (2.15)$$

with

$$\lambda_{\pm} = \frac{1}{2} \left\{ (d-1) \pm \sqrt{(d-1)^2 + 4m^2} \right\}, \quad (2.16)$$

and hereafter $r_0 = 1$. In light of AdS/CFT correspondence, ψ_{\pm} can be interpreted as the expectation value of the operator \mathcal{O}_{\pm} dual to the charged scalar field ψ

$$\psi = z^{\lambda_-} \langle \mathcal{O}_- \rangle + z^{\lambda_+} \langle \mathcal{O}_+ \rangle, \quad (2.17)$$

and the constants μ and ρ are able to be considered as the chemical potential and charge density in the dual field theory. Since the condensation goes to zero ($\psi \rightarrow 0$) near the critical temperature, the Eq. (2.8) reduces to

$$\phi'' + \left\{ \frac{f'}{f} - \left(\frac{d-4}{2n-1} - 2 \right) \frac{1}{z} \right\} \phi' = 0, \quad (2.18)$$

which yields the general solution

$$\phi = \beta + \gamma g(z), \quad (2.19)$$

whose integration constants β and γ are determined by the boundary conditions (2.12), (2.13), (2.14), and (2.15)

$$\phi = \mu, \quad (2.20)$$

i.e. in order to render the gauge field finite near the tip, the Neumann boundary condition near $z = 1$ imposes $\gamma = 0$ so that β is obtained as μ . This means ϕ has only constant solution independent of the power of the Power-Maxwell field n for any dimension d in as the Einstein-Maxwell-scalar theory [34]. Near the origin $z = 0$, one can introduce a trial function $F(z)$ for $\psi(z)$ as in [28]

$$\psi(z)|_{z \rightarrow 0} \sim \langle \mathcal{O}_\pm \rangle z^{\lambda_\pm} F(z), \quad (2.21)$$

which satisfies $F(0) = 1$ and $F'(0) = 0$. Substituting Eqs. (2.20) and (2.21) into Eq. (2.7) we get

$$\begin{aligned} F''(z) &+ \left\{ -\frac{(d-2)z^{d-1}+2}{z(1-z^{d-1})} + \frac{2\lambda_\pm}{z} - \frac{d-4}{z} \right\} F'(z) \\ &+ \left[\frac{\lambda_\pm(\lambda_\pm-1)}{z^2} - \frac{\lambda_\pm}{z} \left\{ \frac{(d-3)z^{d-1}+2}{z(1-z^{d-1})} + \frac{d-4}{z} \right\} \right. \\ &\left. + \frac{\mu^2}{1-z^{d-1}} - \frac{m^2}{z^2(1-z^{d-1})} \right] F(z) = 0, \end{aligned} \quad (2.22)$$

which leads to

$$\left\{ T(z)F'(z) \right\}' - P(z)F(z) + \mu^2 Q(z)F(z) = 0, \quad (2.23)$$

via following functions

$$\begin{aligned} T(z) &= z^{2\lambda_\pm-3}(z^{d-1}-1), \\ P(z) &= -T(z) \left[\frac{\lambda_\pm(\lambda_\pm-1)}{z^2} - \frac{\lambda_\pm}{z} \left\{ \frac{(d-3)z^{d-1}+2}{z(1-z^{d-1})} + \frac{d-4}{z} \right\} - \frac{m^2}{z^2(1-z^{d-1})} \right], \\ Q(z) &= \frac{T(z)}{1-z^{d-1}} \end{aligned} \quad (2.24)$$

After setting the trial function $F(z) = 1 - az^2$, the minimum eigenvalues of μ^2 is calculated from the variation of the following functional [28]

$$\mu^2 = \frac{\int_0^1 dz \left\{ T(z)F'^2(z) + P(z)F^2(z) \right\}}{\int_0^1 dz Q(z)F^2(z)}. \quad (2.25)$$

After taking $m^2 = d(d-2)/4$, from Eq.(2.16) we get the operator \mathcal{O}_- of conformal dimension

$$\lambda_- = \frac{d-2}{2}, \quad (2.26)$$

Then, μ_-^2 is explicitly given by

$$\mu_-^2 = \frac{s_{\mu_-}(a, d)}{t_{\mu_-}(a, d)}, \quad (2.27)$$

where

$$\begin{aligned} s_{\mu_-}(a, d) &= d(d-4) \left\{ (2d-5)(2d-7)(d^3 - 6d^2 + 28d - 24)a^2 \right. \\ &\quad \left. - 2(d-2)^3(2d-7)(2d-3)a + (d-2)^3(2d-5)(2d-3) \right\}, \\ t_{\mu_-}(a, d) &= 4(2d-3)(2d-5)(2d-7) \left\{ (d-2)(d-4)a^2 - 2d(d-4)a + d(d-2) \right\}. \end{aligned} \quad (2.28)$$

When the constant a_- is

$$a_- = \frac{s_{a_-}(d)}{t_{a_-}(d)}, \quad (2.29)$$

$$\begin{aligned} s_{a_-}(d) &= 2d^6 - 11d^5 + 7d^4 + 12d^3 + 132d^2 - 376d + 240 - 2 \left(53d^{10} - 882d^9 \right. \\ &\quad \left. + 6094d^8 - 22310d^7 + 44985d^6 - 43972d^5 + 5624d^4 + 16608d^3 + 12448d^2 \right. \\ &\quad \left. - 33024d + 14400 \right)^{1/2}, \\ t_{a_-}(d) &= 2d^6 - d^5 - 129d^4 + 578d^3 - 620d^2 - 472d + 672, \end{aligned} \quad (2.30)$$

the minimum eigenvalue $\mu_{\min(-)}$ yields

$$\mu_{\min(-)} = \frac{s_{\mu_{\min(-)}}(d)}{t_{\mu_{\min(-)}}(d)}, \quad (2.31)$$

with

$$\begin{aligned} s_{\mu_{\min(-)}}(d) &= \left\{ 11d^5 - 105d^4 + 371d^3 - 600d^2 + 440d - 120 - (d-2) \left(53d^8 \right. \right. \\ &\quad \left. \left. - 670d^7 + 3202d^6 - 6822d^5 + 4889d^4 + 2872d^3 - 2444d^2 - 4656d + 3600 \right)^{1/2} \right\}^{1/2}, \\ t_{\mu_{\min(-)}}(d) &= 2 \left((2d-3)(2d-5)(2d-7) \right)^{1/2}. \end{aligned} \quad (2.32)$$

For example, the minimum eigenvalue μ_{\min} (2.31) for $d = 5$ is given by $\mu_c = \mu_{\min(-)} \approx 0.837$, which is exactly matched with that in [34], and $\mu_{\min(-)} \approx 1.22$ for $d = 6$, and $\mu_{\min(-)} \approx 1.58$ for $d = 7$.

When the scalar field squared mass m^2 is bigger than the Breitenlohner-Freedman bound squared mass $m_{\text{BF}}^2 = -(d-1)^2/4$, the \mathcal{O}_+ is normalizable. Furthermore, since it is possible that the analysis in previous case is applied to any m^2 in the range $m_{\text{BF}}^2 < m^2 < 0$, the chemical potential μ_c is investigated for more general squared mass m^2 . We now deal with operator of the dimension $\lambda_+ = d/2$ before operators of general dimensions.

In the same way in previous case $\mu_{\min(-)}$, taking $m^2 = d(d-2)/4$, the dimension of operator λ_+ (2.16) reduces to $\lambda_+ = d/2$. Then the minimum eigenvalue $\mu_{\min(+)}$ is obtained as

$$\mu_{\min(+)} = \frac{s_{\mu_{\min(+)}(d)}{t_{\mu_{\min(+)}(d)}}, \quad (2.33)$$

with

$$\begin{aligned} s_{\mu_{\min(+)}(d)} &= \left\{ 11d^5 - 33d^4 - 13d^3 + 94d^2 - 60d - d \left(53d^8 - 266d^7 - 114d^6 \right. \right. \\ &\quad \left. \left. + 2558d^5 - 3451d^4 - 4192d^3 + 13804d^2 - 12000d^1 + 3600 \right)^{1/2} \right\}^{1/2}, \\ t_{\mu_{\min(+)}(d)} &= 2 \left((2d-1)(2d-3)(2d-5) \right)^{1/2}. \end{aligned} \quad (2.34)$$

The critical value $\mu_c = \mu_{\min(+)} \approx 1.890$ for $d = 5$ is absolute agreement with the numerical result in [34], and $\mu_{\min(+)} \approx 2.205$ for $d = 6$, and $\mu_{\min(+)} \approx 2.531$ for $d = 7$.

After taking the dimension of operator $\lambda_- = (d-2)/2$ and $\lambda_+ = d/2$, we obtain the critical chemical potential μ_c as the total dimension $d = 5$ to $d = 21$, and so it is linearly proportional to d . We plot these results in Figure 1.

Considering the operators of more general dimensions, the square of chemical potential is obtained as

$$\mu^2 = \frac{s_{\mu^2}(d)}{t_{\mu^2}(d)}, \quad (2.35)$$

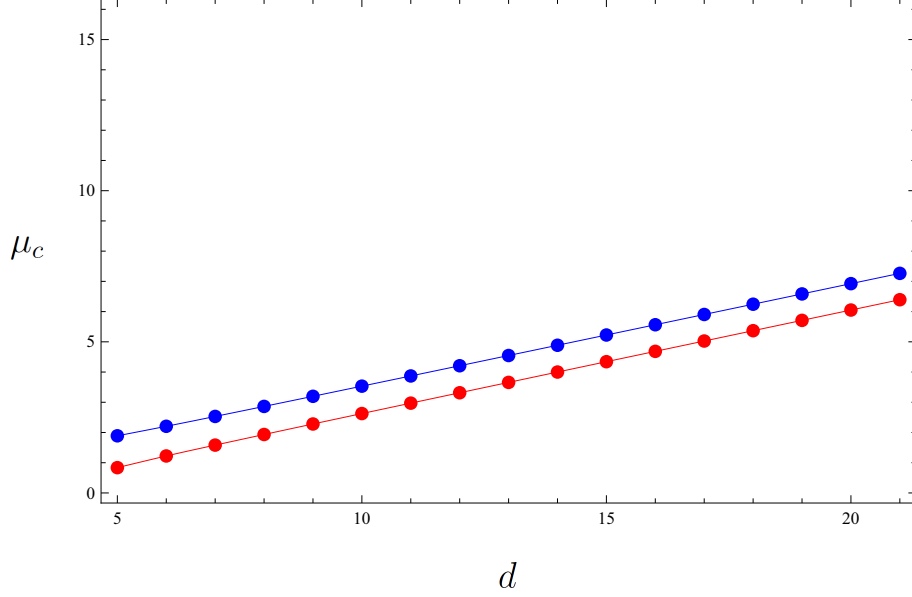


Figure 1: The critical chemical potential μ_c is plotted as the total dimension $d = 5$ to $d = 21$ where red is the dimension of operator $\lambda_- = (d-2)/2$ and blue $\lambda_+ = d/2$.

with

$$\begin{aligned}
s_{\mu_{m^2}}(d) &= \left(\frac{2m^2 + (d-1)\sqrt{(d-1)^2 + 4m^2} + (d-2)(d-7)}{\sqrt{(d-1)^2 + 4m^2} + 2(d-1)} \right. \\
&\quad \left. + \frac{8}{\sqrt{(d-1)^2 + 4m^2} + d-1} \right) a^2 \\
&\quad + 2 \left(\frac{2m^2 + (d-1)\sqrt{(d-1)^2 + 4m^2} + (d-1)^2}{\sqrt{(d-1)^2 + 4m^2} + 2(d-2)} \right) a \\
&\quad + \frac{2m^2 + 2(d-1)\sqrt{(d-1)^2 + m^2} + (d-1)^2}{\sqrt{(d-1)^2 + 2m^2} + 2(d-3)}, \\
t_{\mu_{m^2}}(d) &= \frac{2a^2}{\sqrt{(d-1)^2 + 4m^2} + d+1} + \frac{4a}{\sqrt{(d-1)^2 + 4m^2} + d-1} \\
&\quad + \frac{1}{\sqrt{(d-1)^2 + 4m^2} + d-3}.
\end{aligned} \tag{2.36}$$

In spite of getting the explicit form of the critical potential μ_c , the result is not shown in this article since it is rather lengthy, so we attempt to show the result for $d = 7$ instead. μ^2 for $d = 7$ yields

$$\mu^2 = \frac{s_{\mu_{m^2}}(7)}{t_{\mu_{m^2}}(7)}, \tag{2.37}$$

with

$$s_{\mu_{m^2}}(7) = \left\{ 18m^4 + 786m^2 + 6396 (m^4 + 146m^2 + 2124) \sqrt{m^2 + 9} \right\} a^2 \quad (2.38)$$

$$\begin{aligned} & -2 \left\{ 19m^4 + 855m^2 + 6804 (m^4 + 159m^2 + 2268) \sqrt{m^2 + 9} \right\} a \\ & + 20m^4 + 954m^2 + 7776 (m^4 + 174m^2 + 2592) \sqrt{m^2 + 9}, \\ t_{\mu_{m^2}}(7) & = \left(m^2 + 15 + 5\sqrt{m^2 + 9} \right) a^2 - 2 \left(m^2 + 17 + 6\sqrt{m^2 + 9} \right) a \\ & + m^2 + 21 + 7\sqrt{m^2 + 9}, \end{aligned} \quad (2.39)$$

which leads to the minimum eigenvalue $\mu_{\min(+)}$

$$\mu_{\min(+)} = \frac{s_{\mu_{\min(+)}(7)}{t_{\mu_{\min(+)}(7)}}, \quad (2.40)$$

with

$$\begin{aligned} s_{\mu_{\min(+)}(7)} & = \left[m^8 - 22m^6 - 513m^4 + (8m^6 - 422m^4)\sqrt{m^2 + 9} \right. \\ & + m^2 \left\{ 15198 + 6126\sqrt{m^2 + 9} + 30 \left(5m^8 + 2907m^6 + 200595m^4 \right. \right. \\ & + 3778092m^2 + \sqrt{m^2 + 9}(178m^6 + 28296m^4 + 882612m^2 + 6781536) \\ & + 20344608 \left. \right)^{1/2} - 2\sqrt{m^2 + 9} \left(5m^8 + 2907m^6 + 200595m^4 + 3778092m^2 \right. \\ & + \sqrt{m^2 + 9}(178m^6 + 28296m^4 + 882612m^2 + 6781536) + 20344608 \left. \right)^{1/2} \left. \right\} \\ & - 2 \left\{ 37692 + 12564\sqrt{m^2 + 9} - 255 \left(5m^8 + 2907m^6 + 200595m^4 \right. \right. \\ & + 3778092m^2 + \sqrt{m^2 + 9}(178m^6 + 28296m^4 + 882612m^2 + 6781536) \\ & + 20344608 \left. \right)^{1/2} + 83\sqrt{m^2 + 9} \left(5m^8 + 2907m^6 + 200595m^4 + 3778092m^2 \right. \\ & + \sqrt{m^2 + 9}(178m^6 + 28296m^4 + 882612m^2 + 6781536) + 20344608 \left. \right)^{1/2} \left. \right\} \left. \right]^{1/2}, \\ t_{\mu_{\min(+)}(7)} & = \sqrt{(m^2 - 7)(m^2 - 16)(m^2 - 27)}. \end{aligned} \quad (2.42)$$

We plot the function (2.37) in Figure 2. (a) for $-9 < m^2 < 0$, which indicates that there is always the minimum value of chemical potential squared for various a 's and m^2 's when $a \rightarrow 0$. As squared mass m^2 increases up to the Breitenlohner-Freedman bound squared

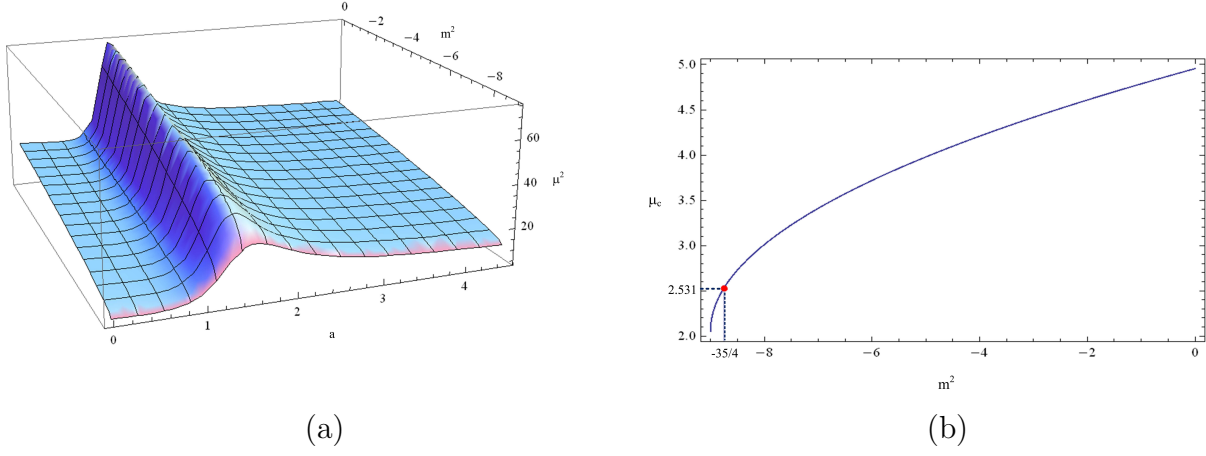


Figure 2: (a) The square of chemical potential μ^2 is plotted as the constant a and the square of mass m^2 for $d = 7$. (b) A plot of the function $\mu_c(m^2)$ for $d = 7$. μ_c has 2.531 when $m^2 = -35/4$.

mass m_{BF}^2 , the critical chemical potential μ_c increases (see in Figure 2. (b)). When μ is very closely located near μ_c , we have

$$\phi'' + \left\{ \frac{f'}{f} - \left(\frac{d-4}{2n-1} - 2 \right) \frac{1}{z} \right\} \phi' = \frac{\langle \mathcal{O}_{\pm} \rangle^2 z^{-4n+2\lambda_{\pm}} F^2(z)}{\alpha n (2n-1) (-1)^{3n+1} 2^n} \frac{\phi}{(\phi')^{2(n-1)} f}, \quad (2.43)$$

by plugging Eq. (2.21) into Eq. (2.8). In such a limit, we may take $\phi(z)$ as

$$\phi(z) = \mu_c + \langle \mathcal{O}_{\pm} \rangle \chi(z), \quad (2.44)$$

where the boundary condition near the tip imposes

$$\chi(z)|_{z \rightarrow 1} = 0. \quad (2.45)$$

Substituting in Eq. (2.43), we obtain

$$\chi'' - \frac{\left(2d(n-1) - 2n + 5 \right) z^{d-1} + d - 4}{(2n-1)(z - z^d)} \chi' = \frac{\langle \mathcal{O}_{\pm} \rangle^{3-2n} z^{-4n+2\lambda_{\pm}} F^2(z)}{\alpha n (2n-1) (-1)^{3n+1} 2^n f} \frac{\mu_c}{(\chi')^{2(n-1)}} \quad (2.46)$$

which for $n = 1$ reduces to

$$\frac{d}{dz} \left[T_1(z) \chi' \right] = - \frac{\langle \mathcal{O}_{\pm} \rangle \mu_c F^2(z)}{2\alpha} \frac{z^{2+2\lambda_{\pm}}}{z^d} \quad (2.47)$$

by introducing the function $T_1(z)$

$$T_1(z) = \frac{z^{d-1} - 1}{z^{d-4}}. \quad (2.48)$$

Considering the operator of dimension $\lambda_- = (d-2)/2$ and taking $\alpha = 1/4$, the above Eq. (2.47) is obtained as

$$\frac{d}{dz} \left[\frac{z^{d-1} - 1}{z^{d-4}} \chi' \right] = -2 < \mathcal{O}_- > \mu_c F^2(z), \quad (2.49)$$

which follows that, under integrating both sides,

$$\begin{aligned} \left. \frac{z^{d-1} - 1}{z^{d-4}} \chi' \right|_0^1 &= \left. \frac{\chi'}{z^{d-4}} \right|_{z \rightarrow 0} = -2 < \mathcal{O}_- > \mu_c z \left(\frac{a^2 z^4}{5} - \frac{2a z^2}{3} + 1 \right) \Big|_0^1 \\ &= -2 < \mathcal{O}_- > \mu_c \left(\frac{a^2}{5} - \frac{2a}{3} + 1 \right). \end{aligned} \quad (2.50)$$

$\phi(z)$ near $z = 0$ is asymptotically given as

$$\phi(z)|_{z \rightarrow 0} \sim \mu - \rho z^2 \approx \mu_c + < \mathcal{O}_- > \left(\chi(0) + \chi'(0)z + \frac{1}{2} \chi''(0)z^2 + \mathcal{O}(z^3) \right), \quad (2.51)$$

which leads to

$$\mu - \mu_c = < \mathcal{O}_- > \chi(0), \quad (2.52)$$

by comparing the coefficients of zeroth order in z in both sides, and from first order we can read

$$\chi'(0) = 0. \quad (2.53)$$

After imposing two boundary conditions (2.45) and (2.53), Eq. (2.51) $\chi(z)$ for $d = 7$ is explicitly obtained as

$$\begin{aligned} \chi(z) = & \frac{< \mathcal{O}_- > \mu_c}{90} \left[-\frac{36}{5} a^2 (z^5 - 1) + 120a(z - 1) - \frac{3}{2} (3a^2 - 10a + 15) \ln(z^4 + 1) \right. \\ & + 2(3a^2 - 10a + 15) \ln(z^3 + 1) + 15\sqrt{2}a \ln(z^2 - \sqrt{2}z + 1) \\ & - 15\sqrt{2}a \ln(z^2 + \sqrt{2}z + 1) + 4\sqrt{3}(3a^2 - 10a + 15) \tan^{-1}\left(\frac{2z-1}{\sqrt{3}}z\right) \\ & + 15(\sqrt{2} - 1) \tan^{-1}(\sqrt{2}z + 1) - 15(\sqrt{2} + 1) \tan^{-1}(\sqrt{2}z - 1) \\ & + 3 \left\{ 3(\sqrt{2}z + 1)a^2 - 10a \right\} - 3 \left\{ 3(\sqrt{2}z - 1)a^2 + 10a \right\} \\ & - \frac{1}{2} (3a^2 - 10a + 15) \ln(2) + 30\sqrt{2}a \coth^{-1}(\sqrt{2}) \\ & \left. - \frac{1}{12} \left\{ 3(9 - 18\sqrt{2} + 8\sqrt{3})a^2 - 10(9 + 8\sqrt{2})a + 15(9 - 18\sqrt{2} + 8\sqrt{3}) \right\} \pi \right] \end{aligned} \quad (2.54)$$

Thus, from Eq. (2.52) we get the qualitative relation between the condensation value $< \mathcal{O}_- >$ and the chemical potential difference $(\mu - \mu_c)$ for arbitrary dimension d

$$< \mathcal{O}_- > \sim \gamma_- \sqrt{\mu - \mu_c}, \quad (2.55)$$

and comparing the coefficients of the z^2 term in (2.51), we read the linear relation between the charge density ρ and $(\mu - \mu_c)$

$$\rho \sim \delta_-(\mu - \mu_c), \quad (2.56)$$

where γ_- and δ_- are positive constants. For example γ_- and δ_- for $d = 5$, $d = 6$, and $d = 7$ are given as

$$\gamma_- = \begin{cases} 1.940 & \text{for } d = 5 \\ 1.987 & \text{for } d = 6 \\ 2.042 & \text{for } d = 7 \end{cases}, \quad \delta_- = \begin{cases} 2.700 & \text{for } d = 5 \\ 4.050 & \text{for } d = 6 \\ 5.399 & \text{for } d = 7. \end{cases} \quad (2.57)$$

Taking $\lambda_+ = d/2$, the Eq. (2.47) is

$$\frac{d}{dz} \left[\frac{z^{d-1} - 1}{z^{d-4}} \chi' \right] = -2 < \mathcal{O}_+ > \mu_c F^2(z) z^2, \quad (2.58)$$

which leads to

$$\left. \frac{\chi'}{z^{d-4}} \right|_{z \rightarrow 0} = -2 < \mathcal{O}_+ > \mu_c \left(\frac{a^2}{7} - \frac{2a}{5} + \frac{1}{3} \right). \quad (2.59)$$

From following the preceding steps, we obtain

$$< \mathcal{O}_+ > \sim \gamma_+ \sqrt{\mu - \mu_c}, \quad \rho \sim \delta_+(\mu - \mu_c), \quad (2.60)$$

where γ_+ and δ_+ are positive constants. For example γ_+ and δ_+ for $d = 5$, $d = 6$, and $d = 7$ are given as

$$\gamma_+ = \begin{cases} 1.801 & \text{for } d = 5 \\ 2.099 & \text{for } d = 6 \\ 2.316 & \text{for } d = 7 \end{cases}, \quad \delta_+ = \begin{cases} 1.329 & \text{for } d = 5 \\ 1.994 & \text{for } d = 6 \\ 2.659 & \text{for } d = 7. \end{cases} \quad (2.61)$$

As Figure 3 shows, supposing the total dimension d more increases than $d = 5$, the coefficient γ_+ in Eq. (2.60) is bigger than the coefficient γ_- Eq. (2.55), and δ_{\pm} increase linearly as d grows up.

We now come back to any power of PM field n , and the Eq. (2.46) leads to

$$\frac{d}{dz} \left[T_n(z) (\chi')^{2n-1} \right] = - \frac{< \mathcal{O}_{\pm} >^{3-2n} z^{-2n+2\lambda_{\pm}} (z^d - 1)^{2n-2} F^2(z)}{\alpha n (-1)^{3n+1} 2^n} \mu_c, \quad (2.62)$$

with

$$T_n(z) = \frac{(z^{d-1} - 1)^{2n-1}}{z^{d-4}}. \quad (2.63)$$

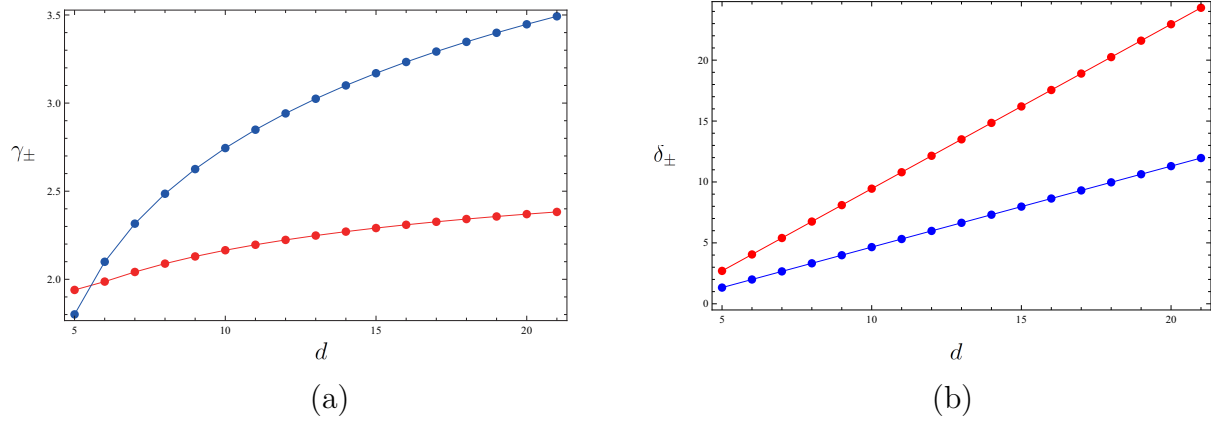


Figure 3: (a) The coefficient γ_{\pm} in Eqs. (2.55) and (2.60) is plotted as the total dimension $d = 5$ to $d = 21$. (b) The coefficient δ_{\pm} is plotted as $d = 5$ to $d = 21$. Here, red is the dimension of operator $\lambda_- = (d - 2)/2$ and blue $\lambda_+ = d/2$.

Then the condensation value $\langle \mathcal{O}_{\pm} \rangle$ and the charge density ρ are qualitatively

$$\langle \mathcal{O}_{\pm} \rangle \sim \xi_{\pm} (\mu - \mu_c)^{\frac{1}{4-2n}}, \quad \rho \sim \zeta_{\pm} (\mu - \mu_c)^{\frac{1}{2-n}}, \quad (2.64)$$

where ξ_{\pm} and ζ_{\pm} are positive constants. It implies that the critical exponent of condensation operator can be changed into $1/(4 - 2n)$ for various n unlike that of Maxwell field.

3 Conclusion

Previous work of the analytical behaviors of the holographic superconductors in five-dimensional AdS soliton spacetime [34] have found that the critical exponent of condensation operator is $1/2$, and the charge density is linearly depending on the chemical potential. We also get the same results in higher-dimensional cases. However, the critical exponent of condensation operator is changed into $1/(4 - 2n)$ in the context of the Einstein-Power-Maxwell field theory, and the charge density is proportional to the chemical potential to the power of $1/(2 - n)$. In addition, since analytical calculations in Eqs. (2.55) and (2.60) indicate AdS soliton background is unstable below the threshold value μ_c but are stable above this value, they may play the role of higher-dimensional insulator and superconductor in the dual field theory, respectively, as in the case of 5-dimensional AdS soliton [33, 34, 35].

References

- [1] M. Born and L. Infeld, Proc. R. Soc. **A 144** (1934) 425.
- [2] B. Hoffmann, Phys. Rev. **47**, 877 (1935).
- [3] W. Heisenberg and H. Euler, Z. Phys. **98**, 714 (1936) [physics/0605038].
- [4] H. P. de Oliveira, Class. Quant. Grav. **11**, 1469 (1994).
- [5] M. Hassaine and C. Martinez, Phys. Rev. D **75**, 027502 (2007) [hep-th/0701058].
- [6] S. S. Gubser, Phys. Rev. D **78**, 065034 (2008) [arXiv:0801.2977 [hep-th]].
- [7] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, Phys. Rev. Lett. **101**, 031601 (2008) [arXiv:0803.3295 [hep-th]].
- [8] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [hep-th/9711200].
- [9] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998) [hep-th/9802109].
- [10] E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998) [hep-th/9802150].
- [11] S. A. Hartnoll, Class. Quant. Grav. **26**, 224002 (2009) [arXiv:0903.3246 [hep-th]].
- [12] C. P. Herzog, J. Phys. A **42**, 343001 (2009) [arXiv:0904.1975 [hep-th]].
- [13] G. T. Horowitz, arXiv:1002.1722 [hep-th].
- [14] E. Nakano and W. -Y. Wen, Phys. Rev. D **78**, 046004 (2008) [arXiv:0804.3180 [hep-th]].
- [15] G. T. Horowitz and M. M. Roberts, Phys. Rev. D **78**, 126008 (2008) [arXiv:0810.1077 [hep-th]].
- [16] G. Koutsoumbas, E. Papantonopoulos and G. Siopsis, JHEP **0907**, 026 (2009) [arXiv:0902.0733 [hep-th]].
- [17] J. Sonner, Phys. Rev. D **80**, 084031 (2009) [arXiv:0903.0627 [hep-th]].
- [18] I. Amado, M. Kaminski and K. Landsteiner, JHEP **0905**, 021 (2009) [arXiv:0903.2209 [hep-th]].

- [19] K. Maeda, M. Natsuume and T. Okamura, Phys. Rev. D **79**, 126004 (2009) [arXiv:0904.1914 [hep-th]].
- [20] H. -b. Zeng, Z. -y. Fan and Z. -z. Ren, Phys. Rev. D **80**, 066001 (2009) [arXiv:0906.2323 [hep-th]].
- [21] O. C. Umeh, JHEP **0908**, 062 (2009) [arXiv:0907.3136 [hep-th]].
- [22] R. Gregory, S. Kanno and J. Soda, JHEP **0910**, 010 (2009) [arXiv:0907.3203 [hep-th]].
- [23] S. S. Gubser, C. P. Herzog, S. S. Pufu and T. Tesileanu, Phys. Rev. Lett. **103**, 141601 (2009) [arXiv:0907.3510 [hep-th]].
- [24] J. P. Gauntlett, J. Sonner and T. Wiseman, Phys. Rev. Lett. **103**, 151601 (2009) [arXiv:0907.3796 [hep-th]].
- [25] R. A. Konoplya and A. Zhidenko, Phys. Lett. B **686**, 199 (2010) [arXiv:0909.2138 [hep-th]].
- [26] X. -H. Ge, B. Wang, S. -F. Wu and G. -H. Yang, JHEP **1008**, 108 (2010) [arXiv:1002.4901 [hep-th]].
- [27] C. P. Herzog, Phys. Rev. D **81**, 126009 (2010) [arXiv:1003.3278 [hep-th]].
- [28] G. Siopsis and J. Therrien, JHEP **1005**, 013 (2010) [arXiv:1003.4275 [hep-th]].
- [29] Y. Brihaye and B. Hartmann, Phys. Rev. D **81**, 126008 (2010) [arXiv:1003.5130 [hep-th]].
- [30] S. Franco, A. Garcia-Garcia and D. Rodriguez-Gomez, JHEP **1004**, 092 (2010) [arXiv:0906.1214 [hep-th]].
- [31] S. Franco, A. M. Garcia-Garcia and D. Rodriguez-Gomez, Phys. Rev. D **81**, 041901 (2010) [arXiv:0911.1354 [hep-th]].
- [32] F. Aprile and J. G. Russo, Phys. Rev. D **81**, 026009 (2010) [arXiv:0912.0480 [hep-th]].
- [33] T. Nishioka, S. Ryu and T. Takayanagi, JHEP **1003**, 131 (2010) [arXiv:0911.0962 [hep-th]].
- [34] R. -G. Cai, H. -F. Li and H. -Q. Zhang, Phys. Rev. D **83**, 126007 (2011) [arXiv:1103.5568 [hep-th]].

- [35] Y. Peng, Q. Pan and B. Wang, Phys. Lett. B **699**, 383 (2011) [arXiv:1104.2478 [hep-th]].
- [36] J. Jing, Q. Pan and S. Chen, JHEP **1111**, 045 (2011) [arXiv:1106.5181 [hep-th]].
- [37] R. -G. Cai, L. Li, H. -Q. Zhang and Y. -L. Zhang, Phys. Rev. D **84**, 126008 (2011) [arXiv:1109.5885 [hep-th]].
- [38] S. Surya, K. Schleich and D. M. Witt, Phys. Rev. Lett. **86**, 5231 (2001) [hep-th/0101134].
- [39] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998) [hep-th/9803131].
- [40] G. T. Horowitz and R. C. Myers, Phys. Rev. D **59**, 026005 (1998) [hep-th/9808079].
- [41] R. Clarkson and R. B. Mann, Class. Quant. Grav. **23**, 1507 (2006) [arXiv:hep-th/0508200].